Master's Thesis

Economic Growth and Output Variability: An Empirical Analysis

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This master's thesis is dedicated to my parents for the many sacrifices they made throughout the years to ensure this day would come to pass.

Abstract

We analyzed the time series experience of twenty-one OECD countries between the years 1961 and 2005. After applying a pooled OLS estimator and running both an outlier-robust parametric regression and a non-parametric regression, the results point to the conclusion that there is strong empirical evidence for a positive relationship between output variability and economic growth. Perhaps our measure of output variability is more suitable for time series of economic growth than ones used previous studies.

Acknowledgements

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In addition to assistance provided by Teresa Lo with editing, I am indebted to a vast number of local and global 'peers' who have contributed towards shaping this work, namely: Michael Sigmund, Dirk Eddelbuettel, Eric Falkenstein, Christian Kraxner, and Markus Pock. Of course, all errors and omissions are mine alone.

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Chapter 1

Introduction

Business-cycle theory and growth theory have traditionally been treated as unrelated areas of macroeconomics; however, this notion has been challenged by recent theoretical models and by empirical evidence that points to long-run performance being explained in part by business-cycle behaviour and output variability. The debate about the direction, if any, and the strength of the link between growth and output variability continues with no sign of resolution within either the theoretical or empirical camp. The purpose of this thesis is to contribute research to the latter. After a brief survey of the topic, we will outline how we approached the research topic.

Black (1981) was one of the first economists who postulated a positive relationship between economic growth and fluctuations in output. According to his theory, the average severity of a society's business cycle is largely a matter of choice. He crafted the following brilliant analogy:

> A business cycle can be likened to a slow motion film of a pebble being washed down a sluice with a rough bottom. The motion of the pebble is erratic, but it keeps returning to the center of the sluice, and it keeps moving down. (Of course, for the business cycle, the film would run in reverse, because output tends to rise over time.) The duration of the pebble's swing will vary greatly. There will be an average time from one swing to the next, but the actual time can differ widely from the average, and the average can change as the construction of the sluice changes. Similarly, business cycles will have some average duration, but the typical cycle will differ greatly from the average of many cycles. ... What happens if we speed up the flow of water in the sluice? The pebble will move more rapidly, and its movements will be more erratic. ... [S]ociety can choose business cycles that correspond to a wide range of water speeds:

> business cycles with slow growth and small fluctuations, or business cycles with fast growth and large fluctuations.

His idea was that economies face a positive risk-return trade-off in their choice of technology, as economic agents would choose to invest in riskier technologies only if the latter were expected to yield a higher return and hence, greater economic growth. Even earlier, Boulding (1966) conjectured that increased uncertainty about future income leads to increased saving. Leland (1968) and Sandmo (1970) were the first to put this idea of precautionary saving on a more formal footing and showed that precautionary saving in response to risk is associated with convexity of the marginal utility function, or a positive third derivative of a von Neumann-Morgenstern utility function. According to Cowen (2006), "The intuition is this: If the effect of your savings is very uncertain, you might either eschew savings altogether ('who knows what it will bring?'), or you might feel a need to save all the more. The third derivative will determine which is the correct decision, and this is not a matter of the discount rate per se." Assuming that more saving leads to more investment, which in turn leads to higher growth, we should observe a positive relationship between growth and volatility as well.

Another line of reasoning supporting a positive link between volatility and growth rests on the Schumpeterian idea (1939) of creative destruction. His view that recessions provide a cleansing mechanism has been revived by several authors (e.g. Hall (1991), Aghion & Saint-Paul (1991), Dellas (1993), and Caballero & Hammour (1994)). Aghion & Saint-Paul (1993) show that the sign of the relation depends on whether the activity that generates growth in productivity is a complement or a substitute to production. In the case where they are substitutes, since the opportunity cost of productivity-improving activities such as reorganizations or training falls in recessions, larger variability leads to higher long-term growth. In the second type of approach, one that relies on learning by doing mechanisms in the tradition of Arrow (1962) or aggregate demand externalities, productivity growth and direct production activities are complements; therefore, a recession has a negative long-run effect on total factor productivity. Another reason why the average growth rate may be reduced by output fluctuations is because they could make returns to investment risker. This would discourage investment and lead to lower economic growth. This view–the importance of entrepreneurial expectations–was emphasized by Keynes (1936).

Bernanke (1983) and Pindyck (1991) suggested that if there are irreversibilities in investment, then increased volatility can lead to lower investment. Other studies postulating a negative effect, especially concerning developing countries, address political instability (Alesina et al. (1992)), credit-market imperfections (Stiglitz (1993)) and sectoral indivisibilities that limit the extent of diversification. (Acemoglu & Zilibotti (1997)). The third possibility is that of no relationship between growth and output variability. This is, for instance, implicit in Friedman's (1968) model of the business cycle in which deviations of the long-run or 'natural' growth rate are due to unstable monetary policy.

The first empirical study was done by Zarnowitz (1981). He identified periods of relatively high and relatively low economic stability by reviewing annual real GDP growth rates in the U.S. between 1882 to 1980 and accounts found in the literature on economic trends and fluctuations. He then calculated the yearly growth rate and the variance of the periods with high economic stability (group A) and low economic stability (group B). Though the mean growth rate of group A was higher, he could not reject the null hypothesis that the difference between the mean growth rates for groups A and B was due to chance.

The first serious econometric study investigating the link between growth, output variability—as measured by the standard deviation of the growth rate—and further macroeconomic variables was conducted by Kormendi & Mequire (1985). By averaging each country's time series experience into a single data point and estimating a cross-section of forty-seven observations, they found that higher output variability leads, *ceteris paribus*, to higher economic growth. Grier & Tullock (1989), who used a pooled structure (five-year averaging) to account for both between- and within-country effects, confirmed Kormendi and Meguire's results.

The next cornerstone of research in this area, an oft-cited body of work, was performed by Ramey & Ramey (1995). Using a panel structure, they measured volatility as the standard deviation of the residuals in a growth regression consisting of the set of variables identified by Levine & Renelt (1992) as the important control variables for cross-country growth regressions. Their model–a special case of an ARCH-in-means model where heteroskedasticity is captured by variation in a country dummy variable–yields a negative relation between long-run growth and volatility.

Caporale & McKiernan (1998) and Grier & Perry (2000) examined the issue from a pure time series perspective. Caporale & McKiernan (1998) ran an ARMA(1,2)-GARCH(0,1)-M model and Grier & Perry (2000) ran a complex bivariate GARCH(1,1)-M model for U.S. GDP growth. The former found a significant positive relationship while the latter found an insignificant positive relationship between growth and volatility.

Mills (2000) applied various filters that are explicitly designed to capture movements in a time series that correspond to business-cycle fluctuations in twenty-two countries. Subsequently, he calculated the standard deviation of the output (filtered) series and visualized the bivariate relationship between growth and volatility by superimposing robust nonparametric curves on scatterplots. He found a positive relationship as well.

Building upon nearly three decades of empirical research, we begin our work by asking what is the appropriate tool for measuring growth volatility. We demonstrate that the widely-used and highly-sophisticated GARCH-in-mean models are inappropriate for this purpose as they require the estimation of too many parameters for the short time series that normally confront economists. We also question the use of band-pass filtering techniques that calculate a country's business cycle and its variance since this approach is based on the assumption of the existence of a more or less rigid (smooth shape, recurring patterns) business cycle.

Our analysis is based on the growth experience of twenty-one OECD countries between 1961 and 2005. After calculating the trend growth rate for each country using the well-known HP-filter, we divided the data for each country into three, fifteen-year, non-overlapping sub-samples. For each sub-sample, the average growth rate and the volatility–based on the squared deviations of the actual growth rate from the trend growth rate–was computed.[1] This not only mitigated the effect of assuming constant volatility and constant growth rates, the technique accounted for the within-country variation of the volatility in our subsequent regression analysis which was based on $3 * 21 = 63$ data points. The finding of our empirical analysis is that there is a significant positive relationship between output variablity and growth. This relationship is robust against outliers and does not hinge on the sub-sample period choosen.

[1] Our measure of volatility is, since it is more stable, superior to another measure of volatility we investigated, namely range-based volatility measures borrowed from securities trading.

Chapter 2

Modelling Volatility

Volatility refers to the spread or dispersion of all likely outcomes of a random variable. It is often measured as the sample standard deviation. Notwithstanding the fact that a measure for the spread of a distribution contains no information on its shape–except when the first two cumulants (mean and variance) are sufficient statistics for the entire distribution–we encounter the following problem when using the sample standard deviation as a measure for volatility: the assumption that both the mean and the standard deviation do not vary with time. In general, nothing can be said about the goodness of the volatility measure without making assumptions about the underlying data-generating process. It should also be noted that whenever one has an unbiased estimator for σ^2, the square root of $\hat{\sigma}^2$ is a biased–depending on the shape of the distribution and the sample size–estimator of σ due to Jensen's inequality.[1]

2.1 Garch-in-Mean Regression Models

In the GARCH-in-Mean (GARCH-M) model the conditional variance of the error term is used as an explanatory variable in the equation for the conditional mean of the variable to be explained:

$$g_t = \kappa + \gamma\sigma_t^2 + x_t'b + u_t \tag{2.1}$$

In finance, g_t could be the return of an asset and a significant parameter γ would support the hypothesis that returns of an asset contain a risk premium that is proportional to the variance of that asset's return.[2] The error term follows a GARCH(p,q) model

[1] $\mathbb{E}[\hat{\sigma}] = \mathbb{E}\left[\sqrt{\hat{\sigma}^2}\right] < \sqrt{\mathbb{E}[\hat{\sigma}^2]} = \sqrt{\sigma^2} = \sigma$

[2] According to financial theory (e.g. the Capital Asset Pricing Model (CAPM)) the risk premium of an asset has to be determined in the context of a portfolio of many assets.

$$u_t = \sigma_t \epsilon_t \tag{2.2}$$

where $\epsilon_t \sim IID(0,1)$ and σ_t^2, the conditional variance of u_t conditional on all the information up to time $t-1$, $\mathcal{F}_{t-1}$, is given as:

$$\mathbb{E}\left[u_t^2|\mathcal{F}_{t-1}\right] = \sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j u_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{2.3}$$

All coefficients in equation 2.3 are necessarily non-negative. Nelson (1990) showed that a GARCH(1,1) process is strictly stationary when $\mathbb{E}[\log(\alpha\epsilon_t^2 + \beta)] < 0$. When $\epsilon_t \sim N(0,1)$, the condition for strict stationarity is weaker then the condition for covariance stationarity $\alpha + \beta < 1$.

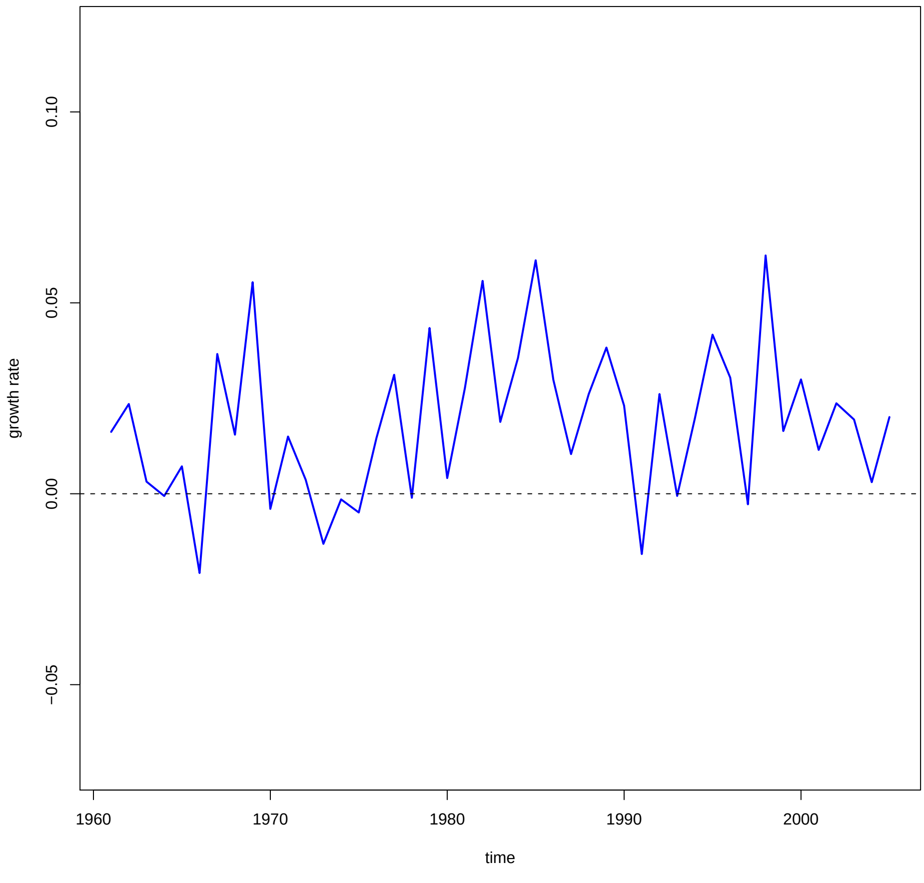

Figure 2.1: Trajectory of a GARCH(1,1)-M process

Figure 2.1 shows a trajectory of a GARCH(1,1)-M process. The risk premium parameter, γ, was set to 2, a value in between those obtained by the

GARCH(0,1)-M model of Caporale et al. (0.7) and the bivariate GARCH(1,1)-M model Grier et al. (3.5). The parameters for the variance equation, α and β, were set to 0.1 and 0.8, respectively. These values are common in finance (see for instance Tsay (2005)) and close to the ones obtained by Grier & Perry (2000) (0.2 and 0.7).[3] Though it seems that such processes are capable of producing series that resemble actual GDP growth rates, unfortunately, very long time series (n » 2500) are required for estimating such processes efficiently.

Running 100 realizations of a GARCH(1,1)-M process with t = 1,...,200 and with the parameters as given above and re-estimating the process[4] yielded the following distribution of the GARCH-in-Mean effect, $\hat{\gamma}$:

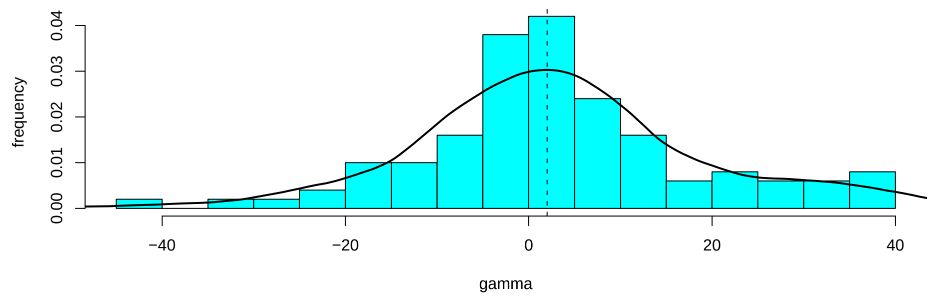

Figure 2.2: Histogram and Empirical Density Function

The average is close to the true mean of our simulation (3 instead of 2) but the standard deviation of 15 is unacceptably large. In 25 percent of our simulation we obtained an estimate for γ that was at least twice as large but had the opposite sign (-4 instead of 2). Apart from this technical obstacle, the implication of the fact that the measure for volatility is based solely on forecast uncertainty seems to be not fully understood when the mean equation 2.1 contains additional regressors.

2.2 Band-pass Filters for Calculating Business Cycles

It is often assumed that the time series under investigation, Y_t, can be represented as a weighted sum of periodic functions of the form $\cos(\omega t)$ and $\sin(\omega t)$ where ω denotes a particular frequency:

[3] The intercepts were set to $\omega = 0.0001$ and $\kappa = 0.005$, respectively and $\epsilon \sim \mathcal{N}(0,1)$

[4] We used R. R uses the Ox package with "garchOxFit" command to estimate GARCH models. Doing the same Monte-Carlo-Simulation with Eviews 5.1 yielded similar results.

$$Y_t = \mu + \int_0^{\pi} \alpha(\omega)\cos(\omega t)\, d\omega + \int_0^{\pi} \delta(\omega)\sin(\omega t)\, d\omega \tag{2.4}$$

An *ideal* band-pass filter is a linear transformation of Y_t that isolates the components that lie within a particular band of frequencies, i.e. the filter only passes frequencies in the range $\omega_L \leq \omega \leq \omega_H$. Applied to GDP growth rates, the filter eliminates very slow-moving ('trend') components and very high-frequency ('noise') components, while capturing intermediate components that correspond to business-cycle fluctuations. The variance of the filtered series, $\hat{g}_t$, could then serve as a measure of volatility.

However, since such an ideal band-pass filter is a moving average of infinite order and therefore requires infinite data, an approximation is necessary for practical applications. Mills (2000) employed the one suggested by Baxter & King (1995) and removed components with frequencies below two years and above eight years.

Building on the graduation method developed by Whittaker (1923) and Henderson (1924), Leser (1961) proposed a filter that is similar to the band-pass, one that has also been widely used in business-cycle research. In economics it is known as the Hodrick-Prescott (henceforth HP) filter. The HP filter is an approximate low-pass filter, i.e. it passes low frequencies but attenuates (or reduces) frequencies higher than the cutoff frequency.

The filtered series is obtained by solving:

$$\min_{\hat{g}_t} \left[\sum_{t=1}^{T} (y_t - \hat{g}_t)^2 + \lambda \sum_{t=2}^{T-1} \left((1-L)^2 \hat{g}_{t+1} \right)^2 \right] \tag{2.5}$$

where $L^n y_t = y_{t-n} \quad \forall n \in \mathbb{N}$. The first summation term in equation 2.5 concerns the fit (squared deviations), the second summation term the smoothness (squares of the second differences) of the filtered series. The parameter λ determines the importance of the smoothness relativ to the fit (trade-off). As $\lambda \to \infty$, $\hat{g}_t$ approaches a linear trend.

The problem with this approach is that, on the one hand, the researcher has some freedom in deciding which band of frequencies to use. On the other hand, the procedure is quite rigid in the sense that it requires the assumed business-cycle has a more or less smooth shape.

2.3 Extreme-Value Volatility Estimators

For years, financial analysts and securities traders have used what has become known as extreme value estimators or range-based estimators in their analysis of price volatility. The simplest, range-based volatility estimator is based on the difference between the maximum and minimum prices observed during a certain period. Parkinson (1980) showed that the daily high-low range $(H_t - L_t)$, properly scaled,

$$\hat{\sigma_P^2} = \frac{1}{4n \ln 2} \sum_{t=1}^{T} (H_t - L_t)^2 \tag{2.6}$$

is also an unbiased estimator of daily volatility – but five times more efficient than the squared daily close-to-close return when the underlying process follows a Brownian Motion. Other, even more effcient estimators that incorporate open, high, low, and close (OHLC) values have been developed (e.g. Garman & Klass (1980) and Yang & Zhang (2000)). The Yang-Zhang estimator[5] is unbiased and independent of process drift and opening gaps.

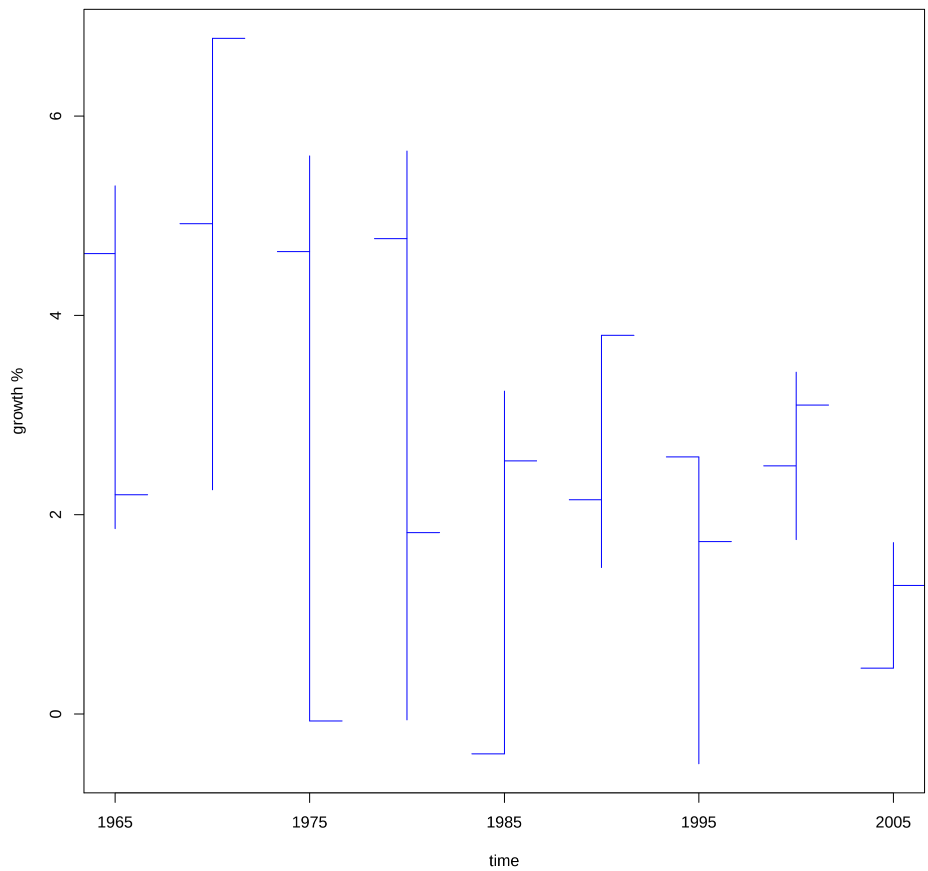

Figure 2.3: OHLC bar for Austria

[5]A linear combination of the classical variance estimator and the extreme value estimator given by Rogers & Satchell (1991)

GDP growth (%)		**OHLC values**	
1961	4.6	Open:	4.6
1962	1.9	High:	5.3
1963	3.4	Low:	1.9
1964	5.3	Close	2.2
1965	2.2		

Table 2.1: GDP growth (%) for Austria

Figure 2.3 is a bar chart. The type of bar displayed is typically referred to as an OHLC bar. Each bar represents data from a certain period in the time series. In this case, each OHLC bar represents five consecutive years of yearly GDP growth rates of Austria (see table 2.1). The value of the first year of each five-year period is the open, while the value of the last year of each five-year period is the close. The value of the high is the highest of each five-year period while the value of the low is the lowest of five-year period. All extreme value volatility estimators use data points from such OHLC bars as input.

The Parkinson estimator uses only two data points from each OHLC bar: high and low. The Yang-Zhang estimator uses all four points from each OHLC bar. We calculated both estimators using only three OHLC bars, i.e. $3 * 5 = 15$ years of data. Since fifteen years of data is required to produce each value, and due to the relatively small number of data points available for analysis, the result is that the first volatility estimates begins in the year 1975. Figure 2.4 shows the Yang-Zhang volatility estimates (dotdashed line), the Parkinson volatility estimates (dotted line), and the volatility estimates produced by the estimator we finally employed (solid line; explained in the next section) from 1975 to 2005 (fifteen-year rolling window). Unfortunately, both volatility series based on the range-based estimators are too volatile for our purpose; however, visual inspection suggests that they pick up the same trend.

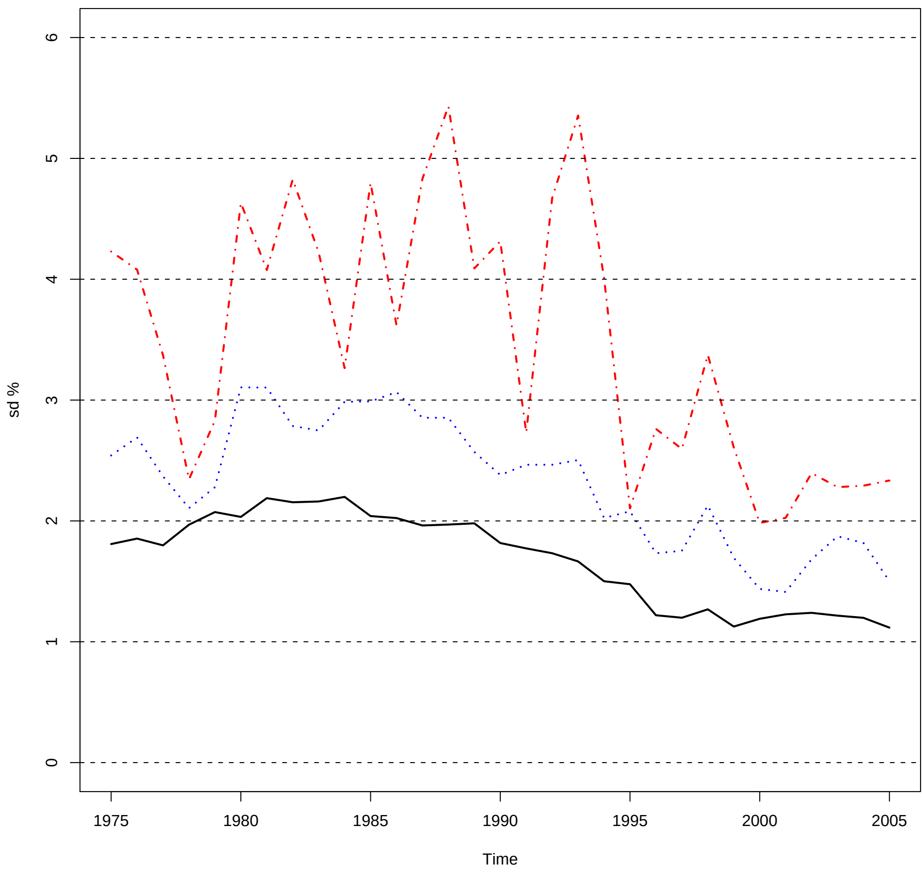

Figure 2.4: Volatility Estimators

Chapter 3

Data and Methodology

3.1 The AMECO Database

The data for this study came from the AMECO database.[1] It is the annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs (DG ECFIN). All 21 countries (Australia, Austria, Belgium, Canada, England, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Luxemburg, Mexico, Netherlands, Portugal, Spain, Sweden, Switzerland, Turkey, and USA) for which continuous annual series for gross domestic product at constant market prices per capita were recorded for the period of 1960-2005 were used for analysis.

3.2 Unit Root Tests

One clear indication that the assumption of a constant mean and a constant variance of a time series cannot be maintained is when unit root tests point to the non-stationarity of the data. In this case, cross-country regressions based on sample mean and sample variance would lead to bogus results.

Testing for unit roots in our series using the standard Augmented Dickey-Fuller[2] (ADF) test—with a constant and a trend in the regression equation—results in the failure to reject the null hypothesis of non-stationarity two-thirds of the time (5 % level of significance). Since the way in which classical hypothesis testing is carried out ensures that the null hypothesis is accepted unless there is overwhelming evidence against it and we want to point out that our series are non-stationary, the appropriate way to proceed is to use a test that has the

[1] http://ec.europa.eu/economy_finance/indicators/annual_macro_economic_database/ameco_en.htm

[2] The number of lags used in the regression is $\text{trunc}\left((\text{length}\,(\text{series}) - 1)^{\frac{1}{3}}\right) = 3$. This corresponds to the suggested upper bound on the rate at which the number of lags should be made to grow with the sample size for the general ARMA(p,q) setup.

null hypothesis of stationarity and the alternative of a unit root. A test with stationarity as null is the KPSS test.

Kwiatkowski et al. (1992) start with the model

$$\begin{aligned} y_t &= \xi t + r_t + \epsilon_t \\ r_t &= r_{t-1} + u_t \end{aligned} \tag{3.1}$$

where $u_t \sim \text{iid}\left(0, \sigma_u^2\right)$, ϵ_t and u_t are independent, and the initial value r_0 is fixed. The ϵ_t satisfy the linear process conditions of Phillips & Solo (1989) (theorems 3.3,3.14) which allow for all ARMA processes, with either homogeneous or heterogeneous innovations.

The test for stationarity in this model is simply

$$H_0 : \sigma_u^2 = 0 \quad \text{vs.} \quad H_A : \sigma_u^2 > 0 \tag{3.2}$$

We performed two tests[3], denoted by KPSS_μ and KPSS_τ based on a regression on a constant μ, and on a constant and a time trend τ, respectively. Even though both tests are very conservative, we reject the stationarity hypothesis in 45% and in 25% of the cases, respectively.

Table 3.1 shows the results for the ADF test and the two KPSS tests for each country. Black squares denote evidence for non-stationarity (ADF: nonrejection of the null hypothesis, KPSS: rejection of the null hypothesis) while white squares denote evidence for stationarity. Out of our sample of 21 countries, all three tests point to stationarity of the data for only five countries.

3.3 Measuring Volatility

We are confronted with the situation whereby some GDP growth series appear to be stationary, while others appear to be trend-stationary, or even nonstationary. In the case of stationarity and trend-stationarity, the growth rate fluctuates around a constant and a linear trend, resepctivelty. In the case of non-stationarity, the growth rate either fluctuates around a deterministic nonlinear trend or a stochastic trend. Using different procedures to calculate the variance for each country could inadvertently result in data mining; therefore, we uniformly applied the same variance-extracting procedure to mantain consistency:

$$\hat{\sigma}^2_{\text{HP}} = \frac{1}{m-1} \sum_{t=1}^{m} (g_t - \hat{\mu}_t)^2, \tag{3.3}$$

where $\hat{\mu}_t$ is the Hodrick-Prescott filtered growth rate that is obtained by solving

[3]To estimate σ_u^2 the Newey-West estimator was used.

Country	**KPSS**$_\mu$	**KPSS**$_\tau$	**ADF**$_\tau$
Australia	□	□	■
Austria	■	□	■
Belgium	■	□	■
Canada	□	□	■
England	□	□	□
Finland	□	□	■
France	■	■	■
Greece	■	■	■
Iceland	□	□	□
Ireland	□	□	■
Italy	■	□	□
Japan	■	■	■
Luxemburg	□	□	■
Mexico	□	□	■
Netherlands	■	□	■
Portugal	■	□	□
Spain	■	■	■
Sweden	□	■	■
Switzerland	□	□	□
Turkey	□	□	□
USA	□	□	□

Table 3.1: Unit Root Tests

$$\min_{\hat{\mu}_t} \left[\sum_{t=1}^{T} (g_t - \hat{\mu}_t)^2 + \lambda \sum_{t=2}^{T-1} \left((1-L)^2 \hat{\mu}_{t+1} \right)^2 \right] \tag{3.4}$$

where $L^n y_t = y_{t-n} \quad \forall n \in \mathbb{N}$. The objective was to set the smoothing parameter such that for both types of stationarity, the filtered series would be a straight line. In case of non-stationarity, the filtered series should display the possible non-linear deterministic trend. Visual inspection (see figure 3.1 to 3.4 suggested setting the smoothing parameter, λ, to 5000. The outcome is in line with our unit-root tests from the previous section. England and the United States are stationary cases par excellence: the growth rate fluctuates around a constant value. Italy is a perfect case of trend-stationarity: the average growth rate has been declining since 1960 at a constant rate. Greece belongs in the nonstationary category: the trend growth rate was declining until the mid-1980s when it reached the bottom and started to increase again.

3.4 Choosing the Regressand and Regressor

Estimating the volatility and the average growth rate over the whole sample and running a cross-country regression afterwards would imply that we assume that both statistics are more or less stable. Visual inspection tells us that this is clearly not the case. Dividing the samples into sub-samples mitigated the effect of assuming constant volatility and constant trend growth rates. Furthermore, we end up with more data points. Of course, there is an upper-bound to the number of sub-samples since we still need enough data points to obtain a 'satisfactory' estimate of the variance (equation 3.3). Since the length of our time series is 45 (1961-2005) we decided to separate them into three (non-overlapping) sub-samples of length 15.[4] The resulting $3*21 = 63$ data points were pooled for our regression analysis. Pooled estimators impose the realistic assumption on our data set that the relationship between regressand and regressor is the same irrespective of whether we are looking across countries or over time within in a country, and that all the errors are drawn from the same distribution.

A valid critique of our analysis is that we do not control for other variables. After including, for instance, inflation as a regressor, the positive effect of volatility on growth becomes even larger. The reason why we refrained from including control variables is that by selectively choosing control variables we would have too much influence on the coefficient under investigation. We think that less damage is done by just assuming that our regressor is orthogonal to the errors.

[4] Splitting the time series into sub-samples of length $\lfloor 45/4 \rfloor = 11$ and of length $\lfloor 45/2 \rfloor = 22$ does not change our main findings, i.e. the relationship between volatility and growth is still positive and statistically significant.

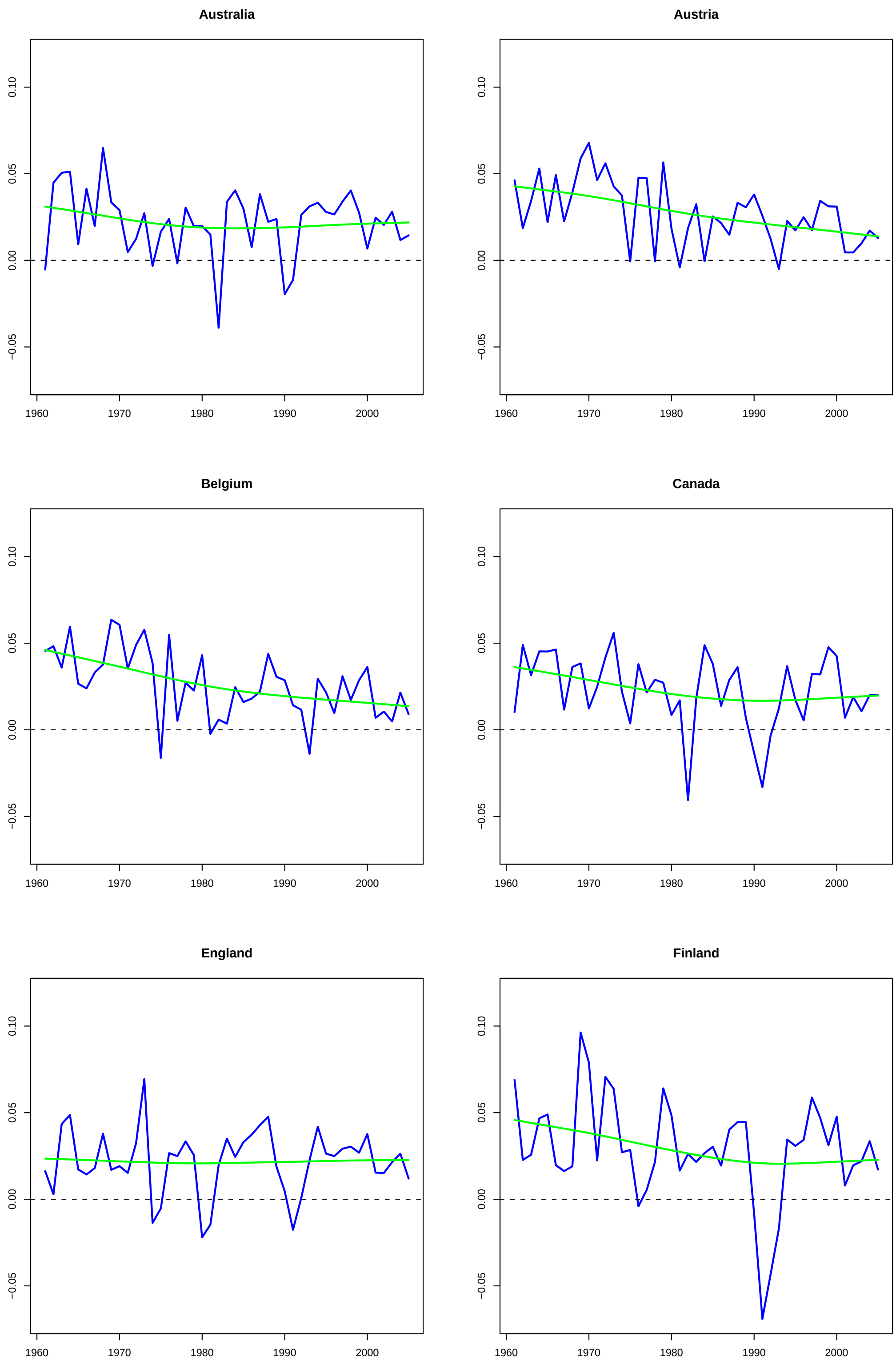

Figure 3.1: Growth Rates of Selected Countries 1

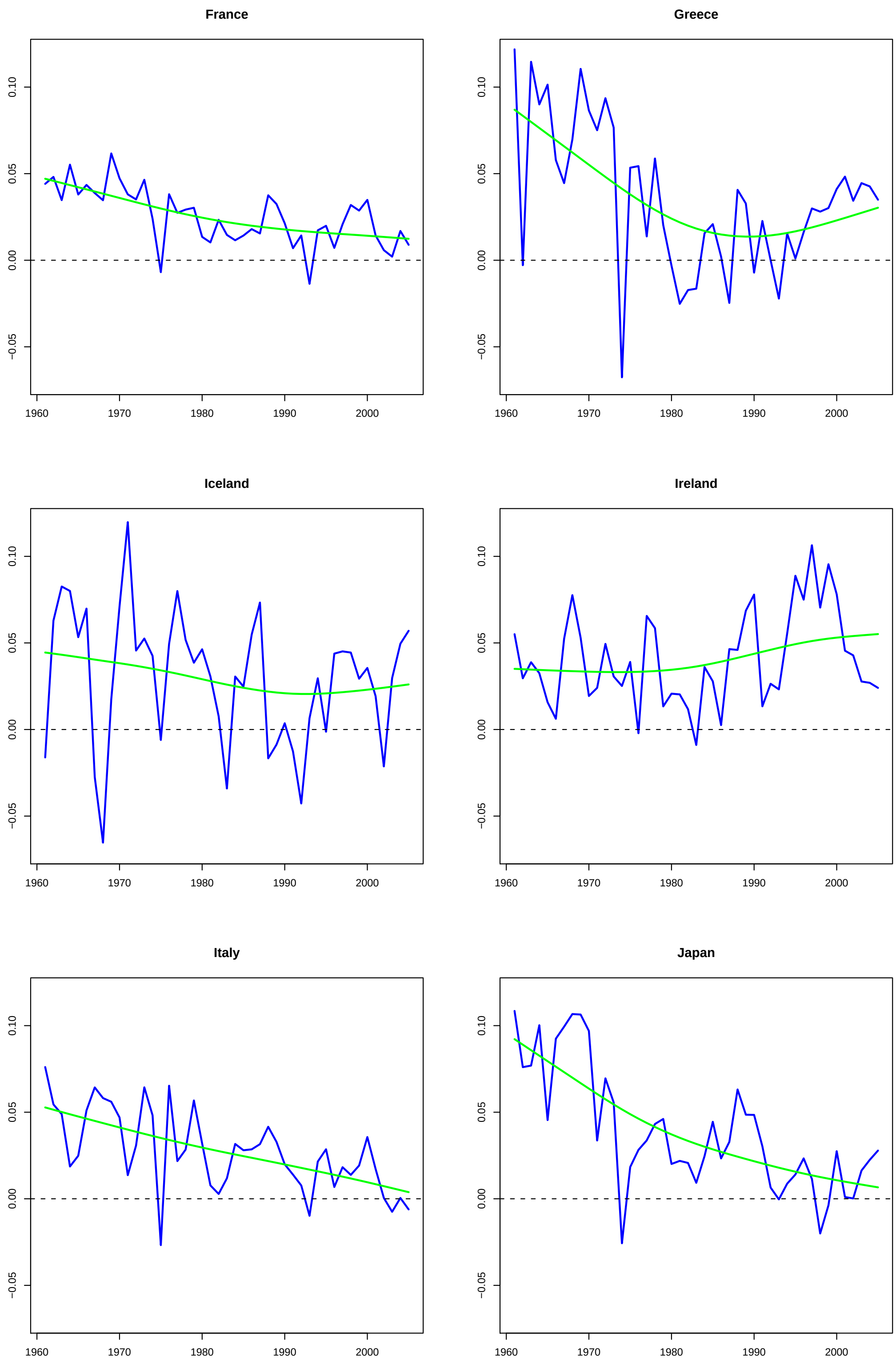

Figure 3.2: Growth Rates of Selected Countries 2

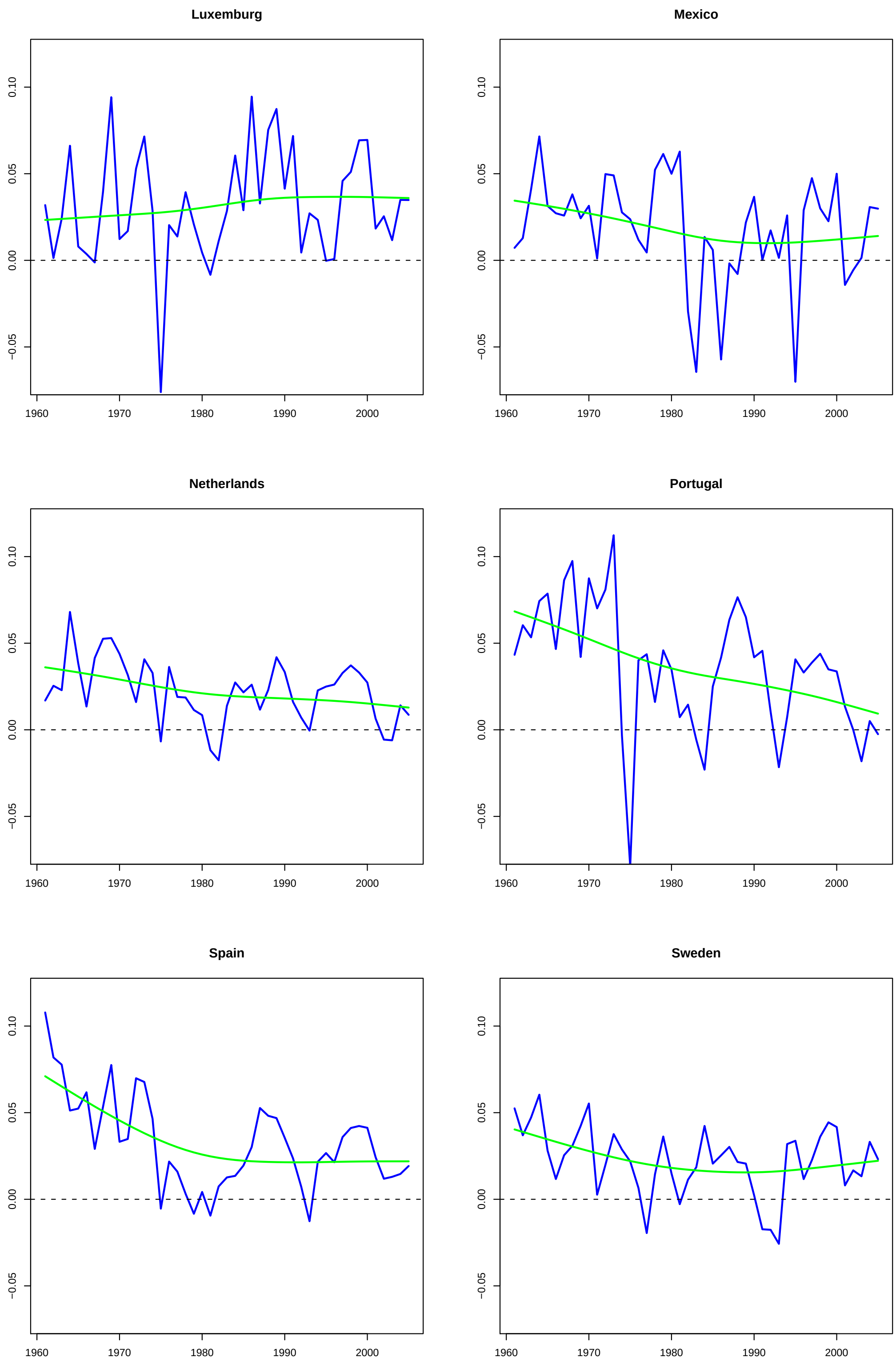

Figure 3.3: Growth Rates of Selected Countries 3

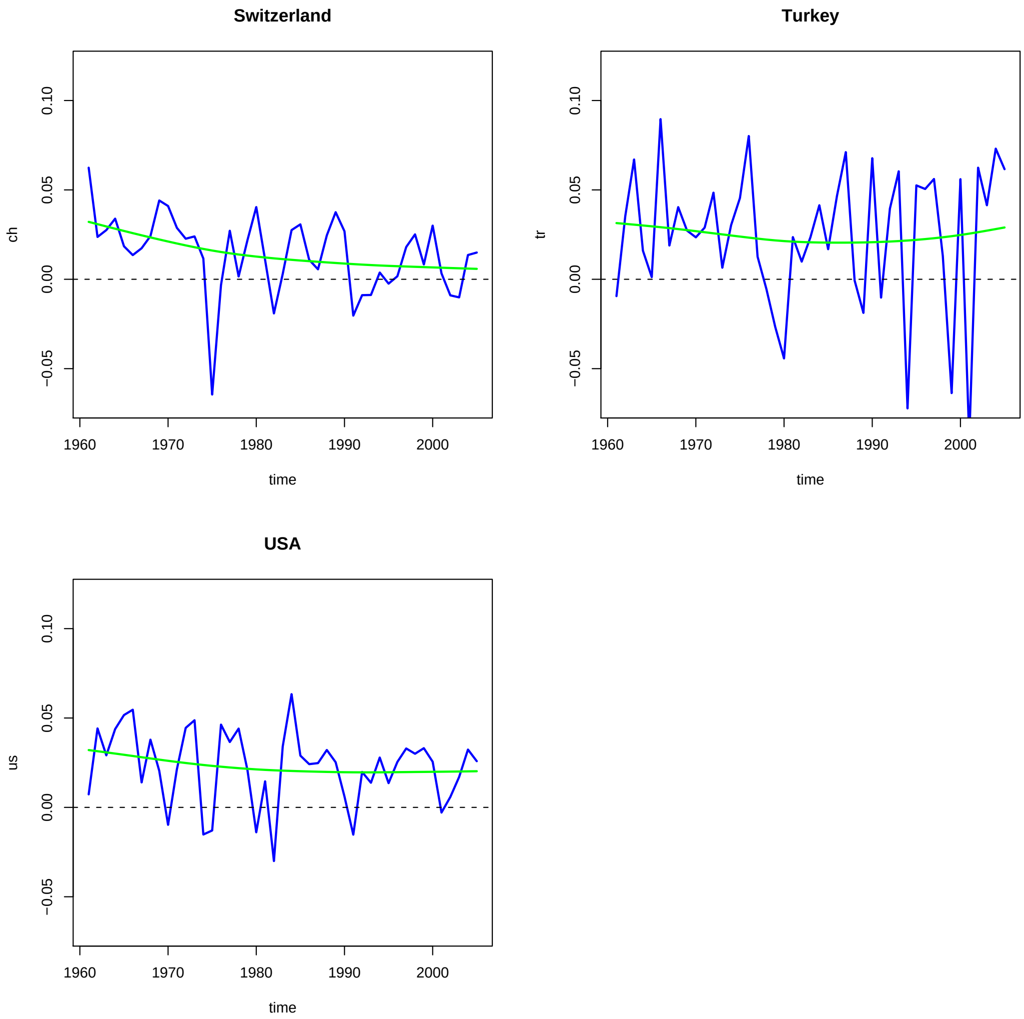

Figure 3.4: Growth Rates of Selected Countries 4

Chapter 4

Empirical Results

We are interested in the functional relationship between the growth rate of GDP, Y, and our measure of its volatility, X. Regression analysis is concerned with the question of how Y can be explained by X. This means a relation of the form

$$\begin{aligned} y_i &= m\left(x_i\right) + \epsilon_i \\ \mathbb{E}\left[Y|X=x\right] &= m(x). \end{aligned} \tag{4.1}$$

where m is a function in the mathematical sense. It determines how the *average* value of Y changes as x changes.

4.1 Ordinary Least Squares

In a parametric approach, the obvious choice is

$$m(x) = \alpha + \beta x \tag{4.2}$$

and functions whose parameters can be estimated by ordinary least squares after applying a linearizing transformation on the variables, like

$$m(x) = \alpha x^{\beta} \tag{4.3}$$

$$m(x) = e^{\alpha + \beta x} \tag{4.4}$$

$$m(x) = \alpha + \beta \ln x \tag{4.5}$$

In equation 4.3 β measures the elasticity[1] of $m(x)$ with respect to x. It can be written as $\ln m(x) = \ln \alpha + \beta \ln x$. In equation 4.4 β gives the proportionate change in $m(x)$ per unit change in x. Vice versa for equation 4.5.

[1]The elasticity measures the percent change in $m(x)$ for a 1 percent change in x. $m(x)_\epsilon = \frac{m'(x)x}{m(x)} = \frac{d \ln m(x)}{d \ln x}$

	$\hat{\alpha}$	s.e.	$\hat{\beta}$	s.e.
Lin-Lin (4.2)	1.5	0.4	0.55	0.15
Log-Log (4.3)	1.7	1.1	0.46	0.13
Log-Lin (4.4)	0.5	0.1	0.17	0.05
Lin-Log (4.5)	1.6	0.3	1.40	0.37

Table 4.1: Regression Estimates

All four models can account for about the same amount of variability in the growth rate (between 15 and 20 percent), with the lin-lin model (4.2, red line) and the lin-log model (4.5, light blue line) coming out leading (see figure 4.1). In both models the estimate for β is significantly different from zero (p-value $<$ 0.001).

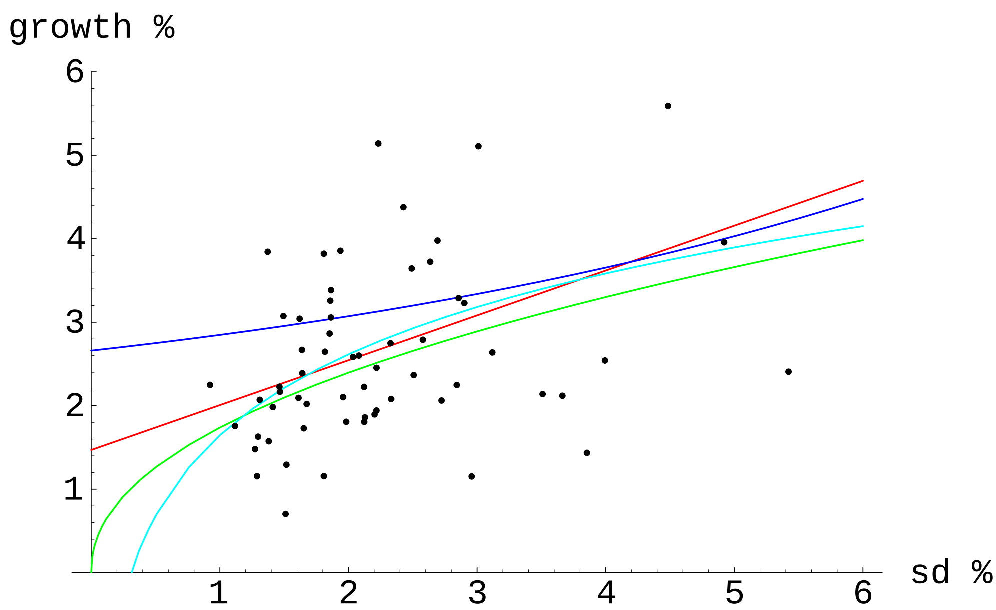

Figure 4.1: Scatterplot and Regression Lines

Unfortunately, the null hypothesis that the residuals come from a normal distribution can be easily rejected. The Anderson-Darling test for normality provides a p-value of 0.03 and the Cramer-von Mises test for normality provides a p-value of 0.02, respectively.[2] Without normality, the ordinary least squares estimator could still be the best linear unbiased estimator (BLUE), but not necessarily the best unbiased estimator (BUE), i.e. $Var(\hat{\beta})$ needn't reach the

[2]The Anderson-Darling test is the recommended empirical distribution function (EDF) test by D'Agostino & Stephens (1986). Compared to the Cramér-von Mises test it gives more weight to the tails of the distribution.

Cramér-Rao lower bound.[3]

4.2 Robust Regression: M-Estimation

A statistical procedure is regarded as 'robust' if it performs reasonablbly well even when the assumption of the statistical model are not true. M-regression, the most common general method of robust regression introduced by Huber (1964), was specifically developed to be robust with respect to the assumption of normality (see Birkes & Dodge (1993)). Consider our linear model

$$y_i = \boldsymbol{x_i'\beta} + \epsilon_i \tag{4.6}$$

for the ith of n observations. The fitted model is

$$y_i = \boldsymbol{x_i'b} + e_i \tag{4.7}$$

The general M-estimator minimizes the *objective function*

$$\sum_{i=1}^{n} \rho(e_i) = \sum_{i=1}^{n} \rho\left(y_i - \boldsymbol{x_i'b}\right) \tag{4.8}$$

where the function ρ gives the contribution of each residual to the objective function. Obviously, for least-squares estimation, $\rho(e_i) = e_i^2$. The Huber M-estimator uses a function ρ that is a compromise between e^2 and $|e|$:

$$\rho(e) = \begin{cases} e^2 & \text{for } |e| \leq \text{k} \\ 2k|e| - k^2 & \text{otherwise} \end{cases}$$

Tukey's biweight estimator is defined as:

$$\rho(e) = \begin{cases} \frac{k^2}{6}\left\{1 - \left[1 - \left(\frac{e}{k}\right)^2\right]^3\right\} & \text{for } |e| \leq \text{k} \\ \frac{k^2}{6} & \text{otherwise} \end{cases}$$

The value k for the Huber-M and Tukey's biweight estimator is called a tuning constant; smaller values of k produce more resistance to outliers, but at the expense of lower efficiency when the errors are normally distributed. We choose the pre-selected values of $k = 1.345\sigma$ for Huber's and $k = 4.685\sigma$ for Tukey's estimator (where σ is the standard deviation of the errors).

Figure 4.2 shows the regression lines for the OLS (red), Huber (blue), and Tukey (green) estimates. Both the Huber and the Tukey estimates of the slope are slightly lower than the OLS estimate, viz. 0.45 and 0.4, respectively, but still significantly different from zero.

[3] The Cramér-Rao lower bound is the reciprocal of the Fisher information of a parameter $\mathbb{I}(\theta)$ and is a lower bound on the variance of an unbiased estimator of the parameter (denoted $\hat{\theta}$).

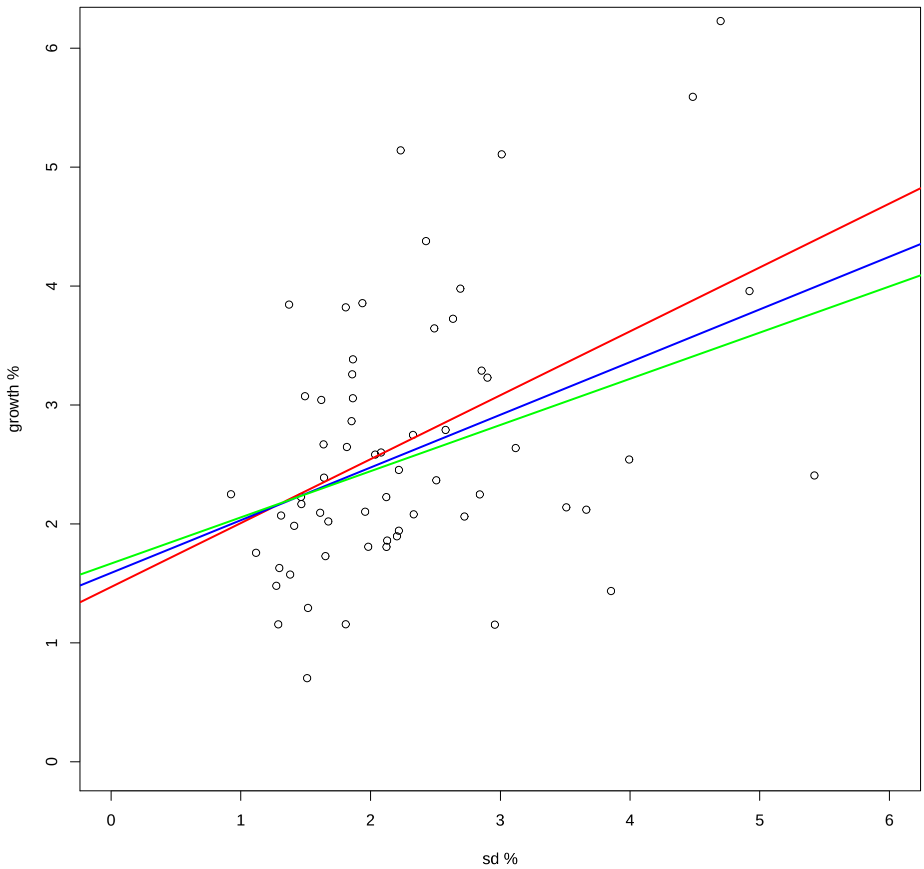

Figure 4.2: OLS, Huber-M, and Tukey's Biweight

4.3 Detection of Influential Data Points

The purpose of any sample is to represent a certain population, actual or hypothetical. Influential data points or outliers[4] in a sample are likely to influence the sample-based estimates of the regression coefficients. There are many sources of outliers such as sampling a member not of that population, bad recording or measurement, errors in data entry, etc. For whatever reason they have come to exist, outliers will lessen the ability of the sample statistics to represent the population of interest. A common method of dealing with apparent outliers in a regression situation is to remove the outliers and then refit the regression line

[4]Hawkins (1980) described an outlier as an observation that 'deviates so much from other observations as to arouse suspicions that it was generated by a different mechanism'. Outliers have also been labeled as contaminants (Wainer (1976))

to the remaining points.

Since no data points that obviously qualify as an outlier could be found by visual inspection, we calculated Cook's distance for each observation. The 100(1-α)% joint confidence region for the parameter vector β is

$$\left(\hat{\beta}-\beta\right)'(X'X)\left(\hat{\beta}-\beta\right) \leq k\hat{\sigma}^2 F_{k,N-k,\alpha} \tag{4.9}$$

Cook's Distance is defined as

$$C_i = \frac{\left(\hat{\beta}-\hat{\beta}_{-i}\right)'(X'X)\left(\hat{\beta}-\hat{\beta}_{-i}\right)}{k\hat{\sigma}^2} \tag{4.10}$$

The 100(1-α)% joint ellipsoidal confidence region for β given in 4.9 is centered at $\hat{\beta}$. The quantity C_i measures the change in the center of this ellipsoid when the ith observation is omitted, and thereby assesses its influence. C_i is the scaled distance between $\hat{\beta}$ and $\hat{\beta}_{-i}$. An alternate form of Cook's distance is

$$C_i = \frac{1}{k}\frac{h_{ii}}{(1-h_{ii})}r_i^2 \tag{4.11}$$

where h_{ii} is the leverage[5] and r_i the studentized residual[6] C_is that are above the threshold value of the $50th$ percentile of the F distribution with k and N-k degrees of freedom (in our case 0.7) are regarded as influential observations. According to this definition, as can be seen in 4.3, our sample does not contain any influential observations.

The most influential data points in our sample are Greece$_{1960-75}$ (#4) with a growth rate of 6.2% and a standard deviation of 4.7%, Turkey$_{1990-05}$ (# 42) with a growth rate of 2.4% and a standard deviation of 5.4%, and Japan$_{1960-75}$ (#46) with a growth rate of 7% and a sd of 3.2%. Running a OLS regression without those three data points yielded a slope of 0.46, wich is the same result as the one obtained by using the Huber-M-Estimator.

4.4 Nonparametric Estimation: Kernel Regression

The nonparametric approach does not assume any functional form for $m(x)$, but rather goes back to the statistical definition of conditional expectation:

[5]The leverage assesses how far away a value of the explanatory variable is from the mean value: the farther away the observation the more leverage it has. h_{ii} is the ith diagonal element of $X(X'X)^{-1}X'$. In the bivariate case $h_{ii} = \frac{1}{n} + \frac{(x_i-\bar{x})^2}{(n-1)s_x^2}$.

[6]The studentized residual is $r_i = \frac{e_i}{s_e\sqrt{1-h_{ii}}}$.

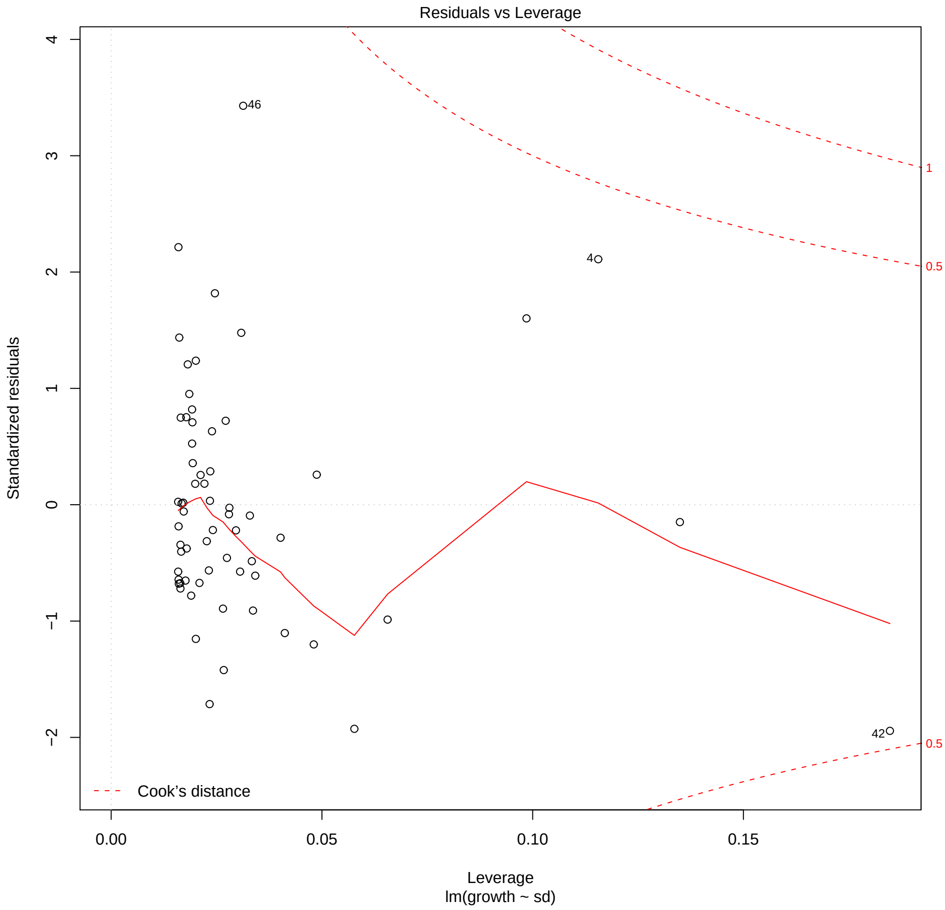

Figure 4.3: Influential Data Points

$$m(x) = \mathbb{E}[Y|X = x] = \int_{-\infty}^{+\infty} y f_{Y|X}(y|x)\, dy = \frac{1}{f_X(x)} \int_{-\infty}^{+\infty} y f_{X,Y}(x,y)\, dy \tag{4.12}$$

Plugging in Kernel estimates for the marginal density, $f_X(x)$, and the joint density, $f_{Y,X}(y,x)$, delivers an estimate $m(x)$ of the conditional expectation at point x:

$$\frac{1}{\hat{f}_X(x)} \int_{-\infty}^{+\infty} y \hat{f}_{X,Y}(x,y)\, dy \tag{4.13}$$

This has become known as the Nadaraya-Watson estimator. Figure 4.4 shows two Nadaraya-Watson regression estimates, one with high bandwith (dark blue line) and one with low bandwith (light blue line). In the dense region, i.e. in the region where many data points are available, the estimates tell the same story as the OLS regression line, so it seems that there really is a linear relationship between volatility and growth. The Nadaraya-Watson estimates become very erratic in the region where the standard deviation is larger than 3.5%. This was to be expected, since only eight data points fall into this region.

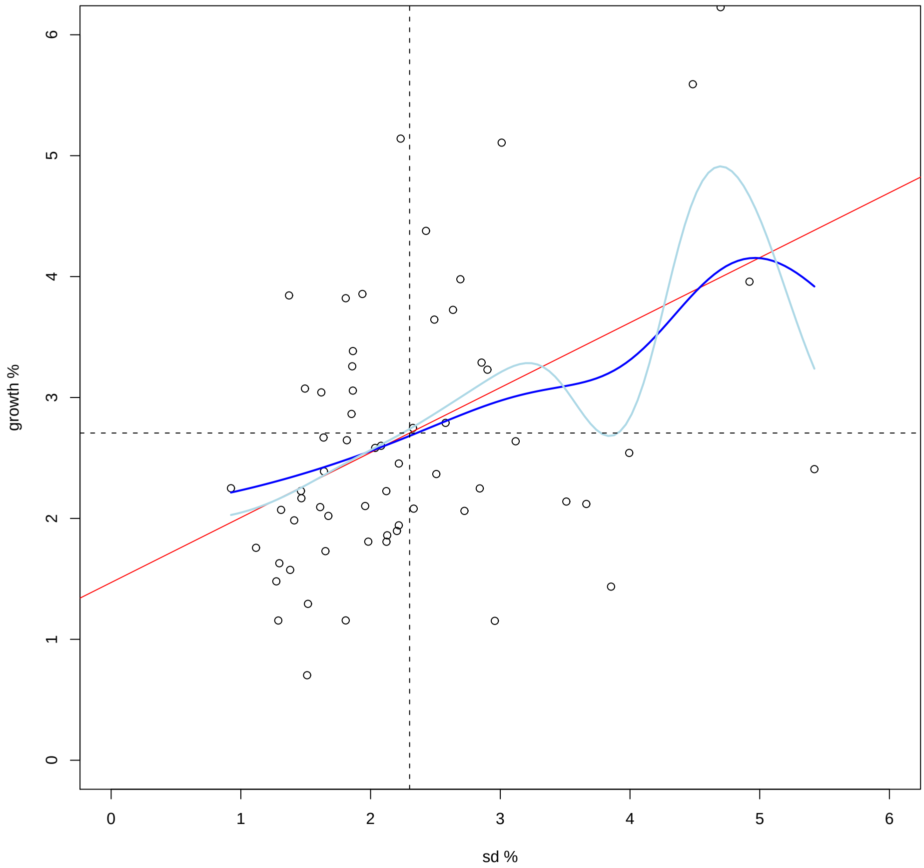

Figure 4.4: Nadaraya-Watson Estimates and OLS Regression Line

Chapter 5

Conclusion

Using the time series experience of twenty-one OECD countries between 1961 and 2005, we have presented strong empirical evidence for a positive relationship between output variability and economic growth. This relationship is robust against outliers and also shows up in a non-parametric setting. A case can be made that our measure of output variability is more suitable than the ones used in previous work for time series of economic growth.

The great Austrian economist Ludwig von Mises once said: "Facts per se can neither prove nor refute anything. Everything is decided by the interpretation and explanation of the facts, by the ideas and the theories." Nonetheless, facts must be ascertained. It should be noted, however, that the purpose of this thesis was solely to explore and examine the relationship between economic growth and its volatility, not having any particular theoretical model in mind. In fact, care should be taken when connecting our results with existing theories. For example, in Fischer Black's model, expected return is related to risk and not the actual return that can be observed. Furthermore, while certain risks that stem from conditions such as political instability partially contribute to higher output volatility, they should not be rewarded. As Sharpe (1998) once said in an interview:

> [T]here's no reason to expect reward just for bearing risk. Otherwise, you'd make a lot of money in Las Vegas. If there's reward for risk, it's got to be special. There's got to be some economics behind it or else the world is a very crazy place.

Since we do not disaggregate volatility in its components, we probably underestimate the strength of the relationship between the risk that Black contemplated and return. Our efforts were directed at finding and applying the most *appropriate tools* to explain the *available data*. We hope our research contributed to the body of knowledge on this subject.

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